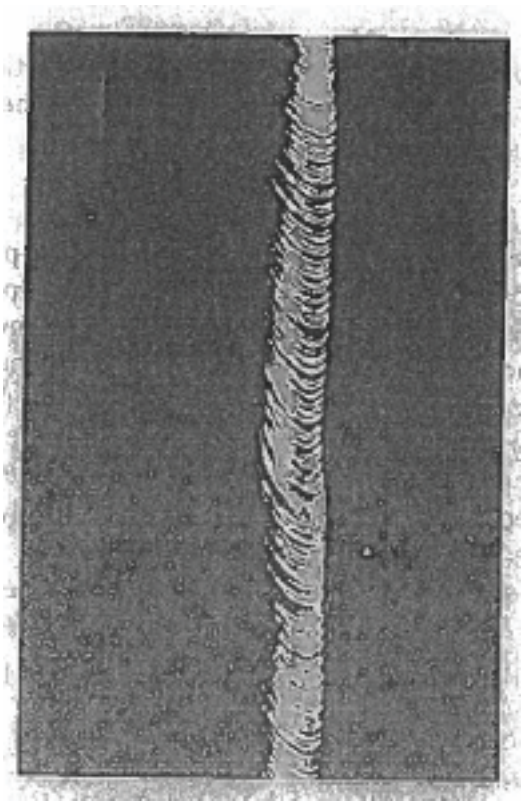
02B_Plasticity: Yield Stress in Perfect crystals



Uniaxial plastic deformation in a single crystal of zinc.

"Slip" at specific crystal planes

(of note: while single crystal is ductile, a polycrystal of zinc shatters into small crystals when hit with a hammer)

Topics:

1. The nature of slip in crystals
2. Derivation of the preferred slip system
3. Crystallography of slip systems

1. The Nature of Slip in Crystals

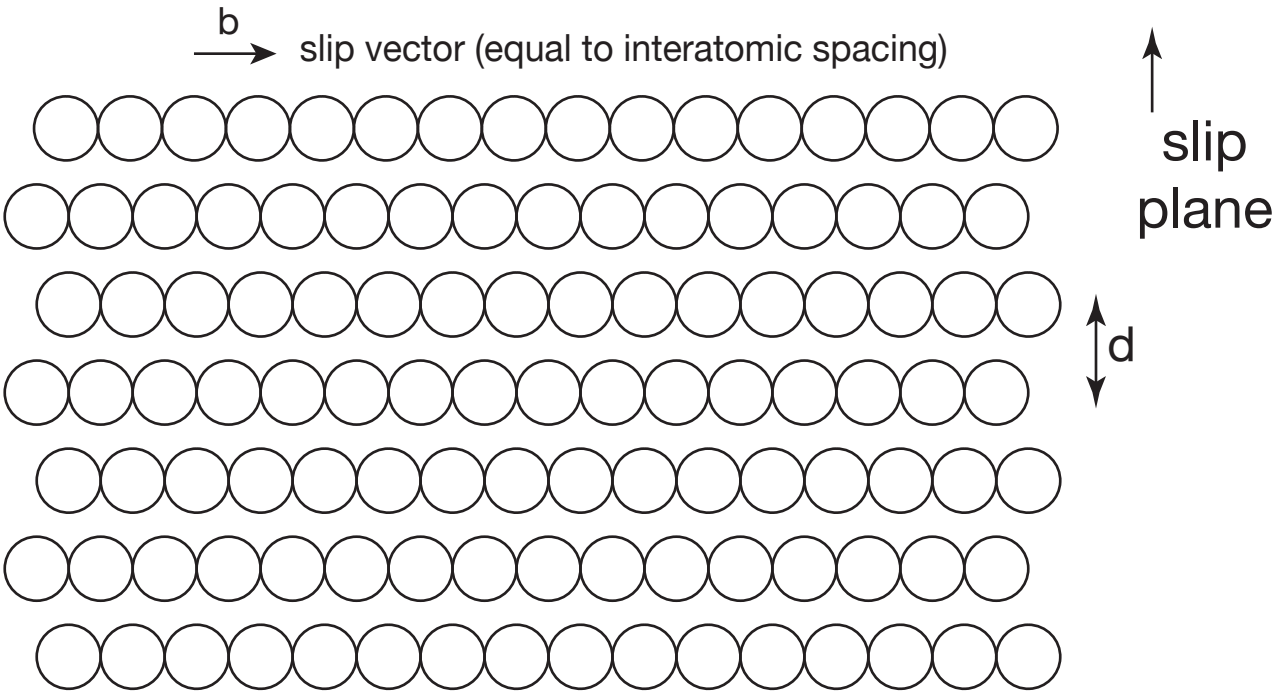
1. The crystal structure is preserved, despite large deformations.

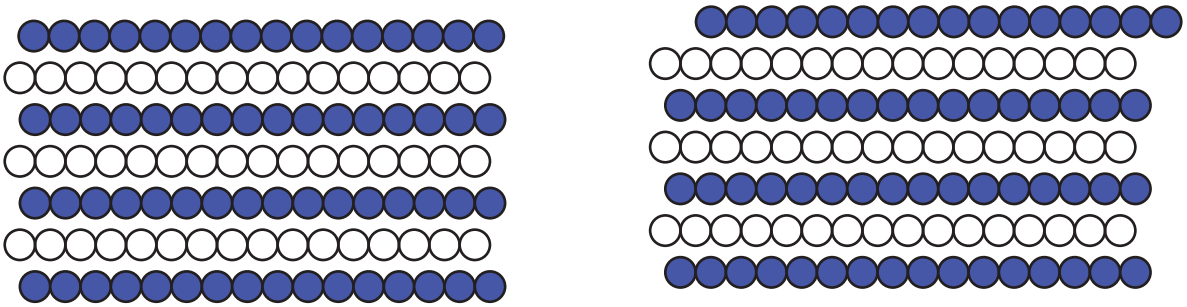
Notes:

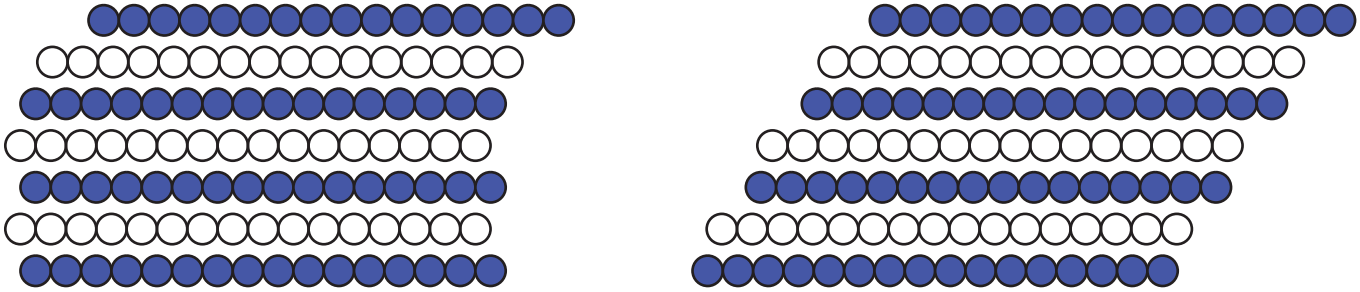
•Slip vector must be a lattice translation vector, so that when slip occurs on a specific crystal plane the structure of the crystal remains unchanged.

(The structure of crystals is determined by X-ray diffraction gives the lattice parameter and the distance between crystallographic planes)

The structure of a crystal is defined by a unit cell which can be repeated in three dimensions to create a physical crystal of any size. The lattice translation vector and the distance between lattice planes are embodied in the unit cell, and it is these properties which are measured by X-ray diffraction.



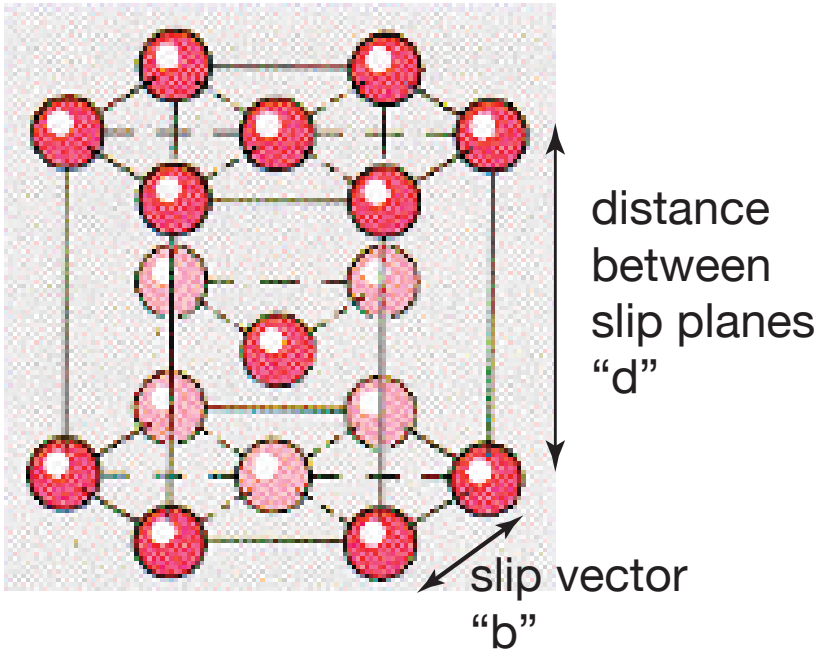




2. Slip vector and slip plane
3. Slip vector must be a lattice translation vector
4. Slip plane must be a crystallographic plane.

The yield stress of the crystal depends on combination of (b,d).

Let us consider the structure of Zn. The unit cell of zinc is as follows,



Slip is possible on different slip systems, meaning different combinations of slip-planes and slip directions.

Each slip system has a different yield stress (i.e. the shear stress required to cause slip).

In the final result: the slip systems which has the lowest yield stress is the one:

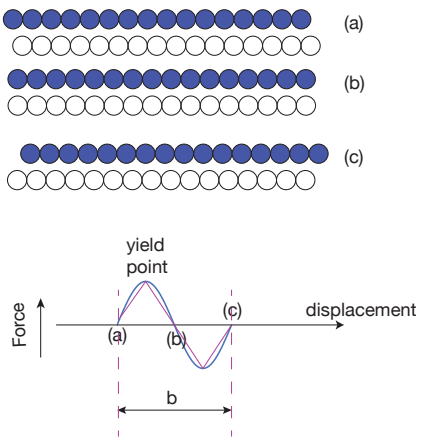
(i) With the shortest slip vector (i.e. the shortest lattice translation vector)

(ii) The largest distance between the slip planes.

Comment: the shortest lattice translation vector would reside in the plane which has the highest packing (density) of atoms. Therefore the crystal planes with largest spacing between them must also have the highest packing.

In summary, the shortest value of "b" and the largest value of "d" will have the lowest yield stress.

2. The Slip System with the Lowest Yield Stress: The Ideal Yield Stress (i.e. the yield stress of a perfect crystal)



Strain at the yield point: $\frac{b/4}{d} = 0.25\frac{b}{d}$

$$\sigma_S = \gamma_S G$$

Strain at yield point is $\gamma_S(\text{at yield point}) = 0.25\frac{b}{d}$

The stress at the yield point, which we call the ideal yield stress because it pertains to a perfect crystal is given by

$$\sigma_S^{ideal} = 0.25\frac{b}{d}G \qquad (1)$$

For example the shear modulus for copper is about 40 GPa. Therefore the ideal yield stress would be 10 GPa.

In reality the yield stress is three or four orders of magnitude smaller. Typically the highest value of the measured yield stress of metals is at the most about 1% of the shear modulus.

$$\sigma_S^{\exp} = 0.01G$$

Nevertheless

$$\sigma_S^{ideal} = 0.25\frac{b}{d}G$$

suggests that the yield stress is lower if "b" is shorter and "d" is greater.

This result from the simple analysis carries forth into real crystals.

That is, the yield stress of real crystals remains guided by Eq. (1) that is the yield stress is lowest for smallest value of "b" and largest value of "d".

The next question is how does the yield stress vary for different crystal systems. We now consider different types of crystal structures

3. Crystallography of Slip Systems

Need a nomenclature for slip vector and slip planes, that is a nomenclature for lattice translation vectors and for crystal planes.

Vectors in a unit cell are given by the notation

[100], [010] and [001]

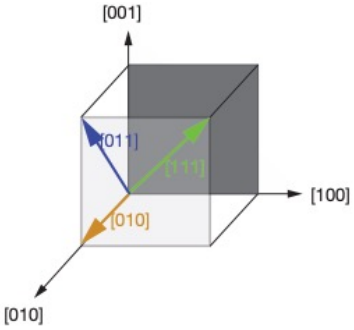
Face diagonal vector is

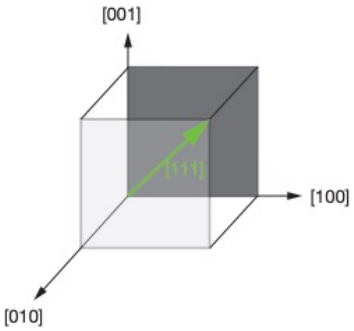
[011], [101], [101], [0-1.1], [-101], [-101]

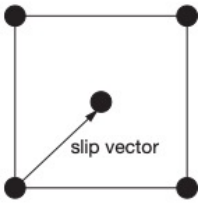
Therefore there are multiplicity of vectors of one type in a crystal. For example here we have six vectors of the type [011]

The system of a given vector are written as

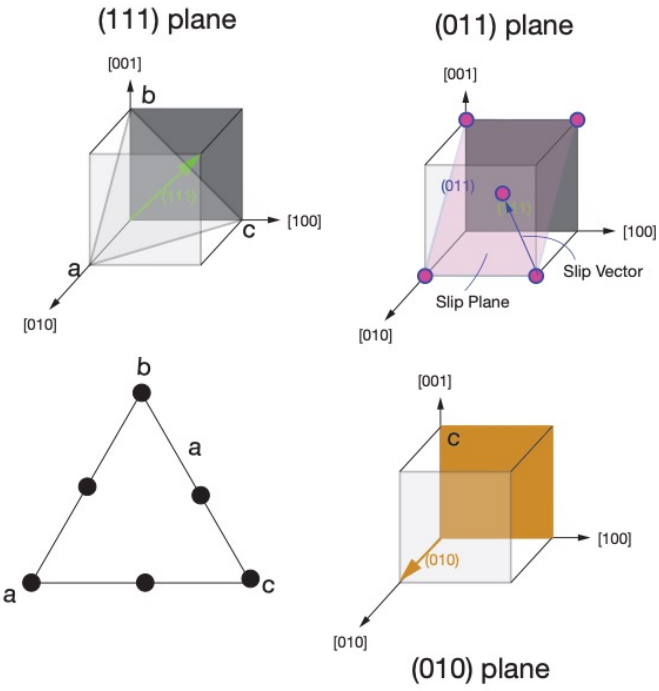
<110>







Planes in a crystal



•The plane is defined by the vector normal to the plane.
•The vector normal to the plane is given by the intercepts of the plane along the axes.

Consider the (111) plane on the right.

The numbers that define this plane are the (inverse of the) intercepts the plane casts on the three cartesian axes.

The system of (111) planes is denoted as {111}

How many planes of can be found,

(111), (-1,1,1), (1,-1,1), (11-1)

Four planes of {111} type in a cubic crystal.

Simlarly {011} planes are defined by the intercept along the three axes (∞,1,1).

(011), (101), etc. (six different planes).

Similarly {100} will give three different planes.

The distance between planes of a given set is calculated by

$d = \frac{1}{\sqrt{1^2+1^2+1^2}}$ for the case of {111} planes.

The distance between the {111} planes is $\frac{a}{\sqrt{3}}$ where "a" is the lattice parameter (the length of the edge of the cube), that is the repeat distance of unit cell.

The above system can be used to find the slip systems with the lowest yield stress, that is those with the shortest "b" and the largest "d".

We can then find multiplicity of slip planes.

The larger the multiplicity number the higher the probability that slip in one crystal can transmit to slip in an adjacent crystal in a polycrystal, which would render a polycrystal more likely to be ductile.

I can ask how many equivalent slip systems are there in {111} <110)

for example

(111) [-110] is a valid slip system since these two vectors are orthogonal to one another (the dot product is zero: -1+1+0=0)

When you consider different combinations, you will find that there are 12 individual slip systems in this set.